A Unified Framework for Posing Strongly-Coupled Multiphysics for Robotics Simulation

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Overview

- We present an optimization-based approach to unify multiphysics via the principle of least action
- Constraints encode physics coupling
- Coupled differential equations derived from action
- Employ variational mechanics to discretize action for simulation

Unified Fluid-Robot Multiphysics

Unified Least Action

External Forces (E.g., Damping)

$$\begin{aligned}\underset{q^f, v^f, q^r, v^r}{\text{minimize}} \quad & \int_{t_0}^{t_1} \mathcal{L}^f\left(q^f, v^f(q^f)\right) + \mathcal{L}^r(q^r, v^r) + F^T q^f\, dt \\ \text{subject to} \quad & \nabla \cdot v^f = 0 \\ & c(v^f, v^r) = 0\end{aligned}$$

Calculus of Variations

Incompressible Navier Stokes

$$\rho \dot{v}^f + \rho (v^f \cdot \nabla) v^f = -\nabla p + \Delta v^f + \frac{\partial c}{\partial v^f}^T \lambda$$

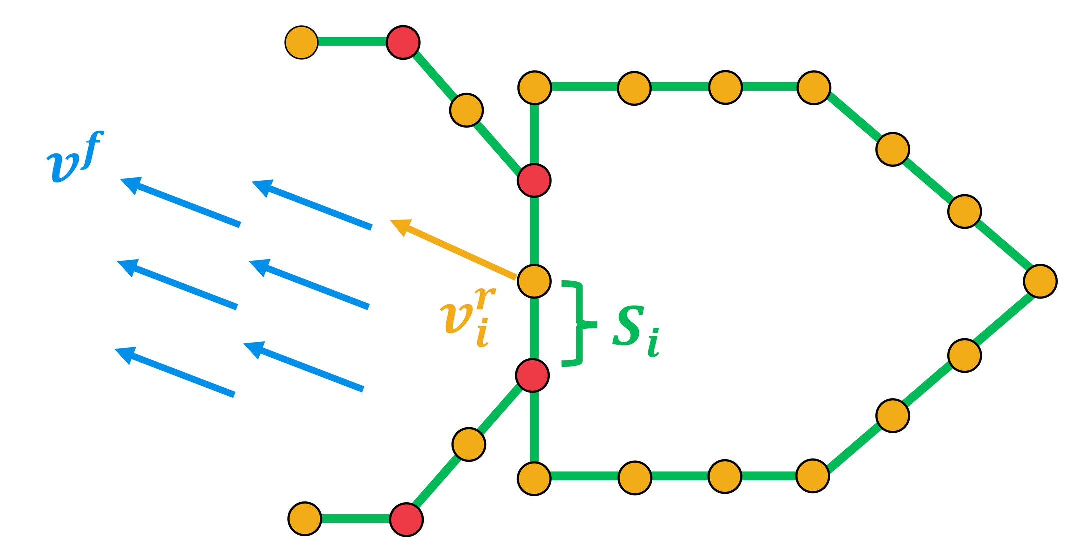
Manipulator Equation

$$M(q^r)\dot{v}^r + C(q^r, v^r) + G(q^r) = \frac{\partial c}{\partial v^r}^T \lambda$$

Coupling Forces

Coupling Constraint

- Keep fluid from penetrating the robot
- At boundary, fluid velocity = robot velocity

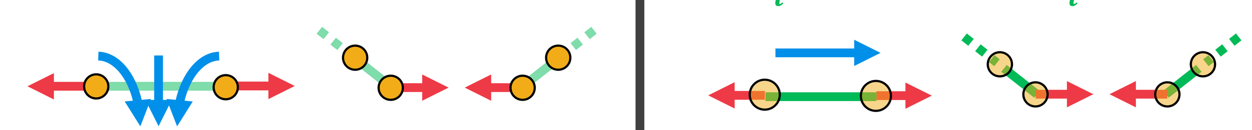


Original [23]

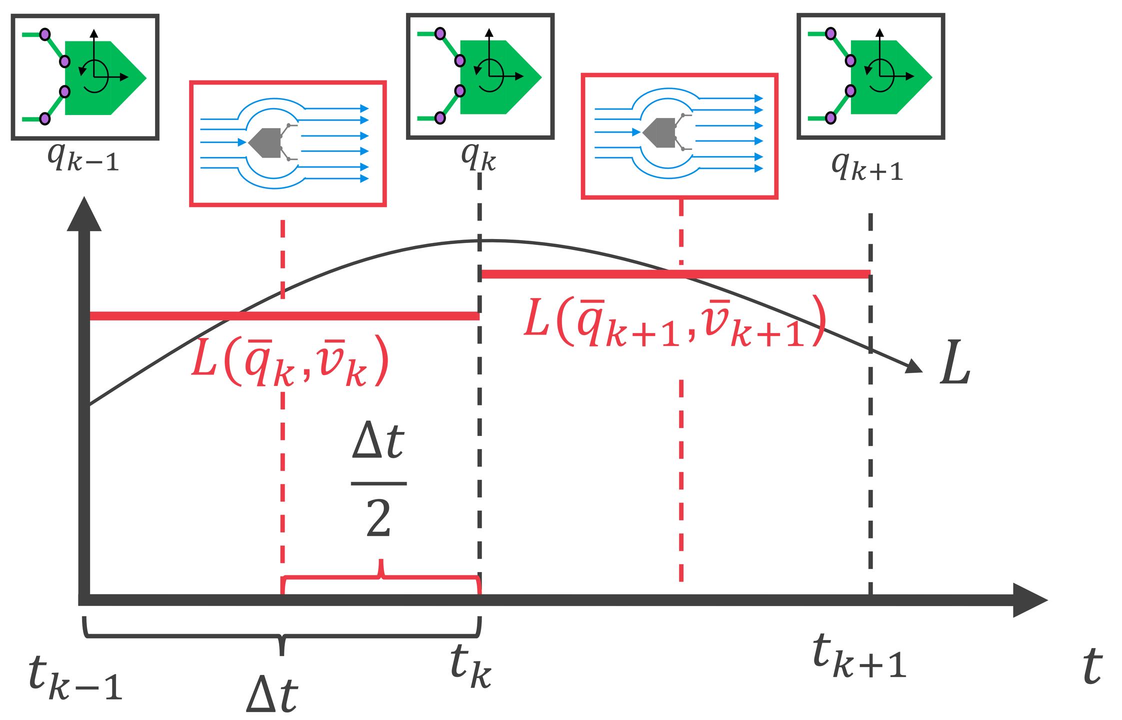
$$E_i v^f = v_i^r$$

Integral-Form (Ours)

$$\int_{S_i} E_i v^f\, ds = \int_{S_i} v^r\, ds$$



Variational Integrator



$$\min_{q, v} \int_{t_0}^{t_1} \mathcal{L}(q, v) + F(t)^T q\, dt \qquad \text{s.t.} \quad c(v) = 0$$

Numerical Quadrature

$$\min_{q_k} \sum \Delta t\, \mathcal{L}(\bar{q}_k, \bar{v}_k) + \Delta t \bar{F}_k^T \bar{q}_k \qquad \text{s.t.} \quad c(\bar{v}_k) = 0$$

First-Order Necessary Conditions

$$\bar{q}_{k+1} = \bar{q}_k + \Delta t \frac{\bar{v}_{k+1} + \bar{v}_k}{2}$$

$$M\bar{v}_{k+1} + \Delta t \frac{\partial c}{\partial v^f}^T \lambda_k = M\bar{v}_k - \Delta t M g + \Delta t \frac{\bar{F}_{k+1} + \bar{F}_k}{2}$$

$$c(\bar{v}_{k+1}) = 0$$

Δt: time step
M: mass matrix
g: gravity

More Info

Sim-to-Real Results